TRINITY COLLEGE

Mathematics Methods U 3,4
Test 2 2022

Section 1 Calculator Free

Applications of Antidifferentiation, Exponential Functions

STUDENT'S NAME _____

DATE: Monday 4th April **TIME:** 35 minutes **MARKS**: 37

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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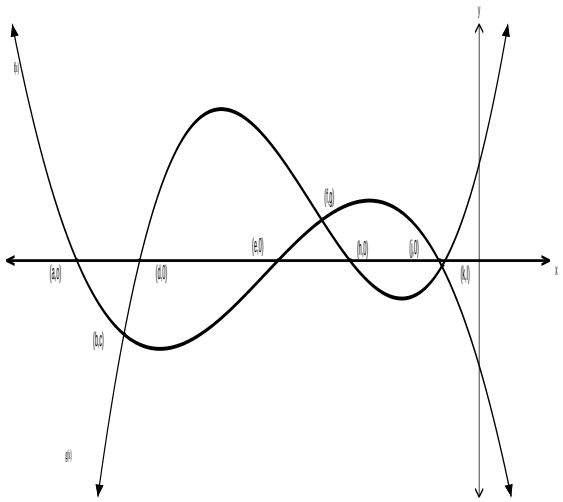
1. (2 marks)

Write an **expression** for the area enclosed between the functions $f(x)$ and $g(x)$.



2. (8 marks)

Find $\frac{dy}{dx}$ for the following:

(a) $y = 5e^{3x}$ [1]

(b) $y = \frac{e^{2x} + e^{x}}{e^{3x}}$ [2]

(c) $y = x^2 e^{x^2}$ [3]

(d) $y = (1 - \cos x)^3$ [2]

3. (9 marks)

Evaluate each of the following definite integrals, leaving your answers as exact values:

(a) $\int_1^{\tan x} e^{t^2}\ dt$ [1]

(b) $\int_{-1}^{1} \frac{3x^2}{4} + \sqrt{x} - 2\ dx$ [3]

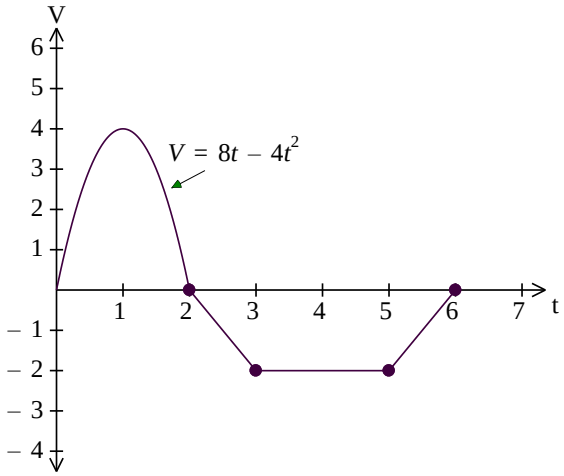
(c) $\int_0^1 (1 + e^x)^2\ dx$ [3]

(d) $\int_1^2 \frac{d}{dx}\left(\frac{1+x}{1+x^2}\right) dx$ [1]

(e) $\frac{d}{dx}(\int_1^x (3t^2 - 5)dt)$ [1]

4. (8 marks)

The graph below represents a velocity – time graph of a body moving in rectilinear motion, where V is measured in metres per second.



(a) Find the distance travelled in the first two seconds. [2]

(b) Find the distance travelled in the first five seconds. [2]

(c) Find the distance from the starting point after five seconds. [2]

(d) Describe, by considering the direction the body is travelling in relation to the origin, how the displacement of the body changes during the period of time shown on the graph. [2]

5. (7 marks)

The function $y = f(x)$ is continuous for all real values of x and $f(x) \geq 0$ for $1 \leq x \leq 4$.

It is known that $\int_1^4 f(x)dx = A$ and $\int_4^6 f(x)dx = -B$ where A and B are positive real numbers.

Find, with reasons, in terms of A and/or B where appropriate:

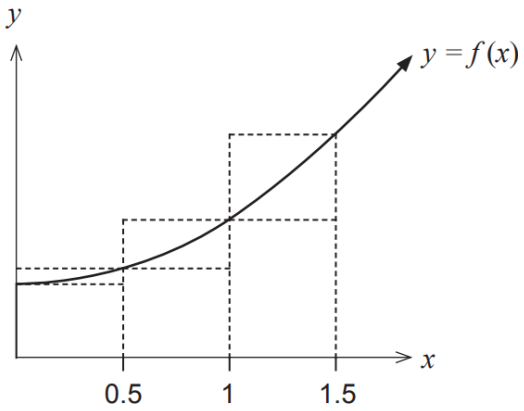
(a) the area of the region trapped between the curve $y = 2\,f(x)$, the x-axis and the lines $x = 1$ and $x = 4$. [2]

(b) $\int_1^6 f(x)\,dx$ [2]

(c) $\int_4^6 2x - f(x)\,dx$ [3]

6. (3 marks)

Consider the function $y = f(x)$ shown graphed below. The table gives the value of the function at the given x values.



x	0	0.5	1	1.5
$f(x)$	20	21	24	29

By considering the areas of the rectangles shown, demonstrate and explain why

$32.5 < \int_0^{1.5} f(x)\,dx < 37.$

TRINITY COLLEGE

Mathematics Methods Unit 3,4
Test 2 2022

Section 2 Calculator Assumed

Applications of Antidifferentiation, Exponential Functions

STUDENT'S NAME _____

DATE: Monday 4th April **TIME:** 15 minutes **MARKS**: 14

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula sheet

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (6 marks)

The rate of change of concentration of a pollutant in a water reservoir can be expressed by $\frac{dC}{dt} = kC$, where C is the concentration in parts per million t days after observations began and k is a constant.

The initial concentration of the pollutant was 82 ppm. Two weeks later this value had dropped to 35 ppm.

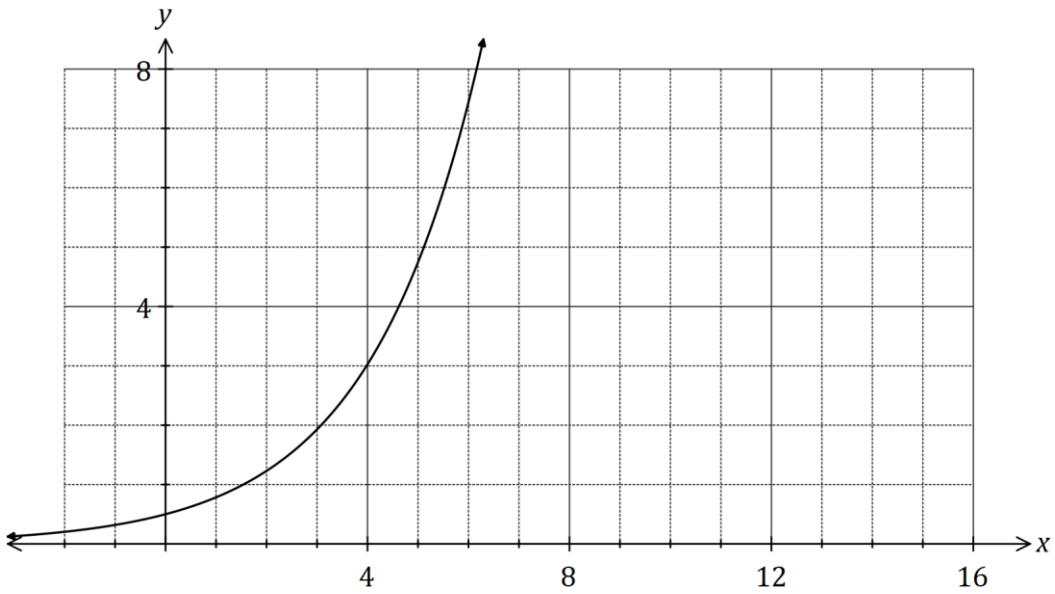
(a) Show that the value of k is -0.0608. [3]

(b) Determine the concentration of the pollutant after three weeks. [1]

(c) The water can be used for drinking once the concentration of the pollutant falls below 5 parts per million.
Determine how long it will take for the concentration to reach this level. [2]

6. (8 marks)

Three functions are defined by $f(x) = 14e^{-0.25x}, g(x) = 0.5e^{0.45x}$ and $h(x) = 0.5$.



(a) One of the functions is shown on the graph above. Add the graphs of the other two functions. [3]

(b) Working to three decimal places throughout, determine the area of the region enclosed by all three functions. [5]